Differentiation Rules and Rates of Change

If f(x) =c, find the derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$f'(x) = \lim_{\Delta x \to 0} \frac{c - c}{\Delta x} = \frac{0}{\Delta x}$

$\lim_{\Delta x \to 0} 0 = 0$

The Constant Rule

The derivative of a constant function f(x)=c is zero

$$\frac{d}{dx}[c] = 0$$

$f'(c) = 0$

Use the definition of the derivative to find the derivative of each function. What patterns do you see? Use your results to find the derivative of $f(x)= x^n$.

a. $f(x) = x^1$ 1 **b.** $f(x) = x^2$ $2x$ **c.** $f(x) = x^3$ $3x^2$

d. $f(x) = x^4$ $4x^3$ **e.** $f(x) = x^{1/2}$ $\frac{1}{2\sqrt{x}}$ $\frac{1}{2}x^{-\frac{1}{2}}$ **f.** $f(x) = x^{-1}$ $\frac{-1}{x^2} = -x^{-2}$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The Power Rule

If n is a rational number, then the function f(x) $=x^n$ is differentiable and

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

x^4

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} =$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

$g(x) = \frac{1}{x^2} = x^{-2}$

$g'(x) = -2x^{-3} = \frac{-2}{x^3}$

Find the slope of the function $f(x)=x^4$ when

1. x=-1 2. x= 0 3. x=1

① Find derivative $f'(x)=4x^3$

② Plug in $4(-1)^3$

m=-4

m=0

m=4

Find the equation of the tangent line to the graph $f(x)=x^2$ at x= -2 (-2, 4)

$f'(x) = 2x'$ ① Derivative

$2(-2)$ ② Plug in x

$\boxed{m=-4}$

③ pt. slope

$y-y=m(x-x)$

$y-4=-4(x-2)$

$y-4=-4x-8$

$\boxed{y=-4x-4}$ eq. of Tangent line

If f is a differentiable function and c is a real number, then cf is also differentiable.

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

c f(x)

$3x^4$

$3(4x^3)$

$f'(x) = 12x^3$

$$g(x) = \frac{2}{x}$$

$2x^{-1} = -2x^{-2}$

$$\frac{-2}{x^2}$$

$$f(x) = \frac{4t^2}{5}$$

$\frac{4}{5}t^2$ $\frac{8}{5}t$

$$\frac{d}{dx}[cx^n] = cnx^{n-1}$$

$$y = \frac{7}{(3x)^{-2}}$$

$7(3x)^2 \rightarrow 7 \cdot 9x^2$

$2 \cdot 63x^{2-1}$

$126x^1$

$\frac{\uparrow\uparrow}{(3x)^2} = \frac{\uparrow\uparrow}{9x^2}$

Sum Rule

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

Difference Rule

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

$\frac{d}{dx}$ The derivative with respect to x

$3x^4 - 2x^3 + x^1 + 10$

$12x^3 - 6x^2 + 1$

Derivative of Sin and Cos

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

y= 2 sinx

$y' = 2(\cos x)$

$y' = 2\cos x$

$\frac{dy}{dx} = 2\cos x$

$$y = \frac{\sin x}{2} = \frac{1}{2}\sin x$$

$y' = \frac{1}{2}\cos x$

$$y = x + \cos x$$

$y' = 1 - \sin x$